U.G. 6th Semester Examination - 2023

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-14

(Statistical Mechanics)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP–A

1. Answer any **five** questions: $2\times5=10$

a) Which statistics (Bose-Einstein or Fermi-Dirac) can be applied to the following particles?

i) alpha particles,

ii) Helium-3 atoms,

iii) Deuterium nuclei,

iv) Neutrons.

b) Show that for a canonical system, mean energy is

$$\langle E \rangle = -\frac{\partial}{\partial \beta}(\ln Z)$$

where Z is the canonical partition function.

c) In how many ways can five Bosons be arranged in three quantum states?

d) Define the term 'degeneracy' in statistical mechanics.

e) Consider distributing two distinguishable particles in the two cells each of volume V/2, calculate entropy S.

f) What is radiation pressure?

g) State Kirchhoff's law of radiation.

h) Prove Rayleigh-Jeans law from Planck's distribution law.

GROUP–B

2. Answer any **two** questions: $5\times2=10$

a) 4.2×10^{21} electrons are confined in a box of volume 1.0 cm^3. Calculate their Fermi wavelength and Fermi energy.

b) Describe the properties of 4He at very low temperature. What is Lambda transition?

c) Prove Stefan-Boltzmann law thermodynamically.

d) Derive expression of Gibbs free energy and enthalpy in terms of partition function Z.

GROUP–C

3. Answer any **two** questions: $10\times2=20$

a) i) Plot distribution function $f(E_i)$ with $(\alpha+\beta E_i)$ for the three statistics.

ii) "Metals can be treated as electron gas" – Explain.

iii) Derive an expression for Fermi energy of the electrons in the metal. 3+2+5

b) i) What is the importance of Sackur-Tetrode equation?

ii) Derive Sackur-Tetrode equation using classical partition function.

iii) What are the limitations of Maxwell-Boltzmann statistics? 2+6+2

c) i) The internal energy and pressure of an ideal gas goes to 0 as $T\rightarrow0$. This is not the case for a degenerate Fermi gas. – Explain.

ii) Plot the variation of density of states g(E) for a Fermionic gas with energy E.

iii) Show that the expression of heat Capacity of a Cold Fermi Gas (metal at low temperature) is $\frac{\pi^3}{3} g(E_F) k_B^2 T$.

Where $g(E_F)$ is the density of states for the three dimensional Fermi gas at Fermi energy E_F, k_B is Boltzmann constant, T is the absolute temperature.

iv) Calculate the probability of finding an electron with energy 6 eV in an electron gas at 1000°C if the Fermi energy of the gas is 5 eV. 2+2+3+3

d) i) Calculate the number of particles in phase space $d^3r d^3p$ obeying Bose-Einstein distribution.

ii) Show that the total number of particles at temperature T in an assembly of completely degenerate boson is given by

$$N = 1.306\sqrt{\pi} CV (k_B T)^{\frac{3}{2}}$$

where $C = \frac{2\pi(2m)^{\frac{3}{2}}}{h^3}$ and V is the volume.

iii) Derive Planck's blackbody radiation formula from BE statistics. 2+3+5
